# WINTER BRAIDS IV
## School on braids and low-dimensional topology

IMB – Université de Bourgogne
10–13 February, 2014

Organizing committee:
P. Bellingeri (Caen), V. Florens (Pau), J.B. Meilhan (Grenoble), E. Wagner (Dijon)

# - Program -

| | Monday 10th | Tuesday 11th | Wednesday 12th | Thursday 13th |
|---|---|---|---|---|
| 9:00 - 9:30 | *Registration* | SALEPCI I | AUDOUX II | CIMASONI III |
| 9:30 - 10:00 | PARIS I | | | |
| 10:00 - 10:30 | | *Coffee Break* | *Coffee Break* | *Coffee Break* |
| 10:30 - 11:00 | *Coffee Break* | PARIS II | SALEPCI II | AUDOUX III |
| 11:00 - 11:30 | CIMASONI I | | | |
| 11:30 - 12:00 | | Guerville | Egsgaard | SALEPCI III |
| 12:00 - 12:30 | Moussard | Bailet | Joergensen | |
| | *Lunch* | *Lunch* | *Lunch* | *Lunch* |
| 14:15 - 14:45 | AUDOUX I | CIMASONI II | PARIS III | Robert |
| 14:45 - 15:15 | | | | Hammarsten |
| 15:15 - 15:45 | Ben Aribi | Magot | Maldonado | Diamantis |
| 15:45 - 16:15 | Korinman | Cohen | Gobet | Goundaroulis |
| 16:15 - 16:45 | *Coffee Break* | *Coffee Break* | *Coffee Break* | |
| 16:45 - 17:15 | Bourrigan | Dalvit | de Miranda e Pereiro | |
| 17:15 - 17:45 | Feller | Cisneros | Mikhalchishina | |

# - Abstracts of Courses -

**Benjamin Audoux** (Marseille)
*The Rasmussen invariant*

Khovanov homology is a link invariant of homological nature which categorifies the Jones polynomial. Jacob Rasmussen extracted a numerical invariant from it which provides a lower bound for the slice genus of knots. After a brief introduction to the categorification of polynomial link invariants, we will present a construction of Khovanov homology. Then, we will extract from it the Rasmussen invariant and use it to give a purely combinatorial proof of the Milnor conjecture on the slice genus of toric knots.

**David Cimasoni** (Genève)
*The geometry of dimer models*

The aim of this minicourse will be to present an introduction to the dimer model to a geometrically minded audience. There will be neither braids nor knots, but I hope to show how several geometrical tools that we know and love (e.g. (co)homology, spin structures, real algebraic curves) can be applied to very natural problems in combinatorics and statistical physics.
More precisely, I plan to cover the following topics:
Lecture 1. Kasteleyn theory for counting the number of perfect matchings on a planar graph.
Lecture 2. Extension of this theory to arbitrary finite graphs.
Lecture 3. The work of Kenyon-Okounkov-Sheffield on the dimer model.

**Luis Paris** (Dijon)
*Non orientable mapping class groups*

In this series of lectures we will adopt the following definition.
Let $M$ be a (non-orientable) surface possible with boundary and non necessarily connected.
We denote by Homeo($M$) the group of homeomorphisms of $M$. The *mapping class group* of $M$, denoted by $\mathcal{M}(M)$, is the group of isotopy classes of elements of Homeo($M$). Here, an isotopy must leave globally invariant the boundary of $M$, but not pointwise. In particular, a Dehn twist along a boundary component is trivial.
This mini-course is an introduction to the subject. It is not intended to experts.
For pedagogical reason, we will focus the presentation of a key result of the domain: the so-called Nielsen-Thurston classification. This theorem says that every element of $\mathcal{M}(M)$ is either of finite order, or pseudo-Anosov, or reducible. Part of the mini-course will be dedicated to explaining the meaning of this theorem, and part will be dedicated to the proof for the non-orientable surfaces.

**Nermin Salepci** (Lyon)
*Lefschetz fibrations*

In this series of 3 talks, we will explore topological and geometrical properties of Lefschetz fibrations, (concentrating mostly on the case of dimension 4).
We will first introduce their topologic properties and present some known classification results.
Then we will talk about their relation to symplectic 4-manifolds and the applications which come with it. We will further continue on investigating their relation to open book decompositions as well as contact structures on 3-dimensional manifolds.

# - Abstracts of Short Talks -

**Pauline Bailet** (Nice)
*On the monodromy of Milnor fibers of hyperplane arrangements*

We describe a general setting where the monodromy action on the first cohomology group of the Milnor fiber of a hyperplane arrangement is the identity.

**Fathi Ben Aribi** (Paris)
*The $L^2$-Alexander invariant detects the unknot*

The $L^2$-Alexander invariant is a knot invariant introduced by Li and Zhang in 2006. It can be seen as a $L^2$-torsion of a certain $L^2$-chain complex derived from the knot complement. It can also be built from the knot group, with Fox calculus, similarly as the Alexander polynomial, except that the operators act on infinite-dimensional Hilbert spaces. In my talk I will present this construction and show that this invariant detects the trivial knot.

**Maxime Bourrigan** (Lyon)
*Braids, signatures and the Blanchfield form*

Through the closure operation, any link invariant defines a function on braid groups. In 2005, motivated by the quest for quasimorphisms on diffeomorphism groups, Gambaudo and Ghys studied the case of the (Levine-Tristram) signatures. More precisely, they gave a formula linking this topological invariant to the Burau symplectic representation.
In fact, link signatures are numerical counterparts of the Blanchfield form, an algebraic invariant describing Poincaré duality in the infinite cyclic cover of a link. In this talk, I will describe how classical results linking the signature of a 4-manifold and the linking form of its boundary extend to this context to give a generalisation of the Gambaudo-Ghys formula.

**Bruno Cisneros** (Dijon)
*The genus of a virtual braid*

Virtual braids are combinatorial generalizations of classical braids that arises from the finite type invariant theory. Usually they are defined via their diagrams, and they can be seen also as non planar graphs, identified up to certain relations. I will present a topological interpretation of these objects as a natural generalization of the classical geometric braids. Finally I will present the genus of a virtual braid.

**Moshe Cohen** (Haifa)
*Dimers from knots*

We consider dimers or perfect matchings on a graph Gamma obtained from a knot diagram. We use these to compute knot polynomial invariants. We also re-cast Kauffman's clock lattice as the graph of perfect matchings of Gamma and give a formula for its height.
Based on joint works with O. Dasbach, H. M. Russell, and M. Teicher.

**Ester Dalvit** (Trento)
*Visualization of welded knots and ribbon 2-knots*

Welded knots are defined diagrammatically via Reidemeister-type moves. Satoh defined a construction to obtain a ribbon torus knot from a welded knot diagram or a 2-knot in the case of a welded arc. The map is known to be surjective but its injectivity remains an open question. In this talk I will tell this story showing movies to visualize the objects involved and their interpretations in various dimensions.

**Carolina De Miranda e Peirero** (Caen)
*The Lower Central and Derived Series of the Braid Groups of the Torus and of the Klein bottle*

We are interested in studying the lower central series and the derived series of the braid groups (resp. pure braid groups) of the torus, $B_n(T)$ (resp. $P_n(T)$), and of the Klein bottle, $B_n(K)$ (resp. $P_n(K)$), one of our aims being to decide whether these groups are residually nilpotent or residually soluble.
For the braid groups of surfaces, these series have been studied in the case of the disc, sphere and the projective plane. Besides that, Bellingeri, Gervais and Guaschi showed that $B_n(T)$ is residually nilpotent if and only if $n \leq 2$.
In this work, we analyse the derived series of $B_n(T)$, and the lower central series and the derived series of $B_n(K)$ and $P_n(K)$. We showed that $B_n(T)$ is residually soluble if and only if $n \leq 4$ and $B_n(K)$ is residually nilpotent if and only if $n \leq 2$.

**Ioannis Diamantis** (Athens)
*Braid equivalence in 3-manifolds with rational surgery description*

In this talk we describe braid equivalence for knots and links in a $3$-manifold M obtained by rational surgery along a framed link in $S^3$. We first prove a sharpened version of the Reidemeister theorem for links in M. We then give geometric formulations of the braid equivalence via mixed braids in $S^3$ using the L-moves and the braid band moves.
We finally give algebraic formulations in terms of the mixed braid groups $B_{m,n}$ using cabling and the techniques of parting and combing for mixed braids. We also provide concrete formulae of the braid equivalence in the case where $M$ is a lens space, a Seifert manifold or a homology sphere obtained from the trefoil.

**Jens Kristian Egsgaard** (Aarhus)
*The Jones representations of braid groups at q = -1 (Part I)*

We show that the Jones representation of $B_n$ evaluated at $q = -1$ is equivalent to the action on a particular quotient of an exterior power of the homology of a ramified double cover of the $n$-punctured sphere via the Birman-Hilden theorem. This generalizes a discovery by Kasahara made for the sphere with six punctures.

**Peter Feller** (Bern)
*The signature of positive braids is linearly bounded by their genus*

We provide linear lower bounds for the signature of positive braids in terms of their length. This yields linear bounds for the topological 4-ball genus of knots that are closures of positive braids.

**Thomas Gobet** (Amiens)
Temperley-Lieb algebras and Zinno's basis

The images of the simple elements of the dual braid monoid of type A in the Temperley-Lieb algebra form a basis of it as shown by Zinno. We want to understand the coefficients of the change base matrix between that basis and the diagram basis.

**Dimos Goundaroulis** (Athens)
*Framization of the Temperley-Lieb Algebra*

In this talk we propose a framization of the Temperley-Lieb algebra. The framization is a procedure that can briefly be described as the adding of framing to a known knot algebra in a way that is both algebraically consistent and topologically meaningful. Here, our framization is defined as a quotient of the Yokonuma-Hecke algebra. The main theorem provides necessary and sufficient conditions for the Markov trace defined on the Yokonuma-Hecke algebra to pass through to our framization. Finally, we discuss the possible knot invariants that can be defined through this procedure.

**Benoit Guerville** (Pau)
*A topological invariant of line arrangements*

In a joint work with E. Artal Bartolo and V. Florens, we construct a topological invariant of line arrangements - linked with Alexander's module and characteristic varieties. It is based on the inclusion of the boundary manifold in the complement, and it is simply computable. It is a strong invariant since it allows to detect new examples of Zariski pairs ; that is a pair of arrangements with the same combinatorics but with different topologies. It can be viewed as an analogy of linking matrix of a link.

**Carl Hammarsten** (Washington)
*Heegaard Floer Homology and Branched Spines*

A *3*-dimensional closed manifold *Y* represented by its branched spine has a canonical Heegaard decomposition. We present this decomposition graphically in the form of a Strip Diagram. We show that strip diagrams have nice properties which greatly simplify the calculation of Heegaard Floer homology for "most" manifolds. Motivated by this work, we present a combinatorial definition of a chain complex which we expect to be homotopically equivalent to the Heegaard Floer one, yet significantly smaller. Finally, we consider the presentation of a branched spine by its O-graph and show how to reformulate our definition in these terms.

**Soeren Fuglede Joergensen** (Uppsala)
*The Jones representations of braid groups at $q = -1$ (Part II)*

We discuss a conjecture by Andersen, Masbaum, and Ueno stating that for all large enough $k$, the level $k$ quantum representation of a given pseudo-Anosov mapping class has infinite order. As an application of the results described in the first part of the talk, we prove this conjecture for a large family of pseudo-Anosov mapping classes of punctured spheres.

**Julien Korinman** (Grenoble)
*Decomposition of quantum representations*

I will describe representations of the mapping class group of surfaces arising in TQFTs associated to gauge groups *U(1)*, *SU(2)* and *SO(3)* and present their decomposition into irreducible factors.

**Miguel Maldonado** (Caen)
*Mapping class groups and coverings*

We consider the punctured mapping class group of a surface and analyze the relation between this group and that of its covering surface. The case for non-orientable surfaces is emphasized.

**Jean-Mathieu Magot** (Grenoble)
*Combinatorial review of the relation* $\Theta = \lambda_{CW} + p_1/4$

The Θ invariant of parallelized rational homology 3-spheres was first defined by M. Kontsevich in the nineties, as a certain count of embeddings of the Theta graph.
G. Kuperberg, D. Thurston and C. Lescop showed that this invariant is equal to $\lambda_{CW} + p_1/4$, where $\lambda_{CW}$ is the Casson-Walker invariant and $p_1$ is an invariant of parallelizations defined as a relative Pontrjagin class.
In this talk, we present an alternative, purely combinatorial proof of this result. The proof builds on recent works of C. Lescop, who gave a combinatorial formula from Heegaard diagrams for the $\Theta$ invariant, and relies on the theory of finite type invariants.
Our talk will review all prerequisites.

**Yuliya Mikhalchishina** (Novosibirsk)
Local Representations of braid groups

The local linear representations of braid group $B_3$ are under investigation as well as the local homogeneous representations of braid group $B_n$, $n \geq 2$. The connection of these representations with the Burau one is under study. Using the Wada representations of $B_n$ in the automorphism group Aut($F_n$) of a free group the linear representations of $B_n$ are constructed.

**Delphine Moussard** (Pisa)
*Equivariant triple intersections*

Given a null-homologous knot $K$ in a rational homology sphere $M$, and the standard infinite cyclic covering $\widetilde{X}$ of *(M,K)*, we define an invariant of triples of curves in $\widetilde{X}$, by means of equivariant triple intersections of surfaces. We prove that this provides a map Φ on A^{\otimes 3}, where A is the Alexander module of *(M,K)*, whose isomorphism class is an invariant of the homeomorphism class of the pair *(M,K)*. For a fixed Blanchfield module (A,b), *i.e.* an Alexander module A endowed with a Blanchfield form b, we consider pairs *(M,K)* whose Blanchfield modules are isomorphic to (A,b), equipped with a fixed isomorphism from the Blanchfield module of *(M,K)* to (A,b). In this setting, we compute the variation of Φ under borromean surgeries, and we describe the set of all maps Φ.

**Louis-Hadrien Robert** (Strasbourg)
*Grothendieck groups of Khovanov-Kuperberg algebras*

The $sl_3$ homology is a variant of the Khovanov homology. The construction starts with $sl_3$ instead of $sl_2$. The geometrical counterpart of the $sl_3$-homology involves foams rather than surfaces.
The Khovanov homology, and the $sl_3$ homology, have a version for tangles. It involves some algebras called $H_n$ in the first case and $K^{\varepsilon}$. the projective indecomposable modules over these algebra decategorify on dual-canonical bases. While in the $sl_2$, this modules are very easy to identity, in the $sl_3$ case it is much more difficult (and still open).