# WINTER BRAIDS VI
## School on braids and low-dimensional topology

Université de Lille I
22-25 February, 2016

Organizing committee
P. Bellingeri, A. Bodin, V. Florens, J.B. Meilhan, E. Wagner

## - Programme -

*The meeting will be held in "Turing amphitheatre", building M3, first floor.*
*Registration and coffee break will be held in "Room Duhem", building M3, first floor.*

<table>
<tr><th></th><th>Monday 22th</th><th>Tuesday 23th</th><th>Wednesday 24th</th><th>Thursday 25th</th></tr>
<tr><td>9h00-9h30</td><td>Registration</td><td rowspan="2">Kashaev I</td><td rowspan="2">Borodzik III</td><td rowspan="2">Popescu-P. III</td></tr>
<tr><td>9h30-10h00</td><td rowspan="2">Sakasai I</td></tr>
<tr><td>10h00-10h30</td><td>Coffee break</td><td>Coffee break</td><td>Coffee break</td></tr>
<tr><td>10h30-11h00</td><td>Coffee break</td><td rowspan="2">Sakasai III</td><td rowspan="2">Kashaev II</td><td rowspan="2">Kashaev III</td></tr>
<tr><td>11h00-11h30</td><td rowspan="2">Borodzik I</td></tr>
<tr><td>11h35-12h05</td><td rowspan="2">Borodzik II</td><td>Morvan</td><td>Moussard</td></tr>
<tr><td>12h10-12h40</td><td>Ben Aribi</td><td>Stylianakis</td><td></td></tr>
<tr><td>Lunch</td><td rowspan="2"></td><td rowspan="2"></td><td></td><td rowspan="2"></td></tr>
<tr><td>14h00-15h00</td><td>Poster session</td></tr>
<tr><td>15h00-16h00</td><td>Sakasai II</td><td>Popescu-P. I</td><td>Popescu-P. II</td><td></td></tr>
<tr><td>16h00-16h30</td><td>Coffee break</td><td>Coffee break</td><td>Coffee break</td><td></td></tr>
<tr><td>16h30-17h00</td><td>Kohli</td><td>Damiani</td><td>Celoria</td><td></td></tr>
<tr><td>17h00-17h30</td><td>Detcherry</td><td>Conway</td><td>Chavli</td><td></td></tr>
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# - Abstracts -

## Mini courses

**Maciej Borodzik** (Warsaw University)

*Heegaard Floer homology with a view towards geometry of complex plane curves*

The lecture will be an introduction to Heegaard--Floer homology focused on applications in algebraic geometry. I will give definitions of the Heegaard-Floer homology, but the exposition will be mostly based on examples, rather than on giving rigorous definitions. The tentative plan is as follows.

1. Introduction. A few exemplary complexes are presented and their properties are given. A formal definition of knot Floer homology and Heegaard Floer homology will is given.

2. I will define d-invariants in Heegaard Floer homology and show their basic applications. I will show how to calculate d-invariants of large surgeries on some families of knots.

3. Two applications in algebraic geometry will be given. First one will be the proof of semicontinuity of semigroups and the second one will be the proof of the conjecture of Fernandez de Bobadilla, Luengo, Melle-Hernandez and Nemethi. Some further generalizations will be shown.

**Rinat Kashaev** (University of Geneva)

*Triangulations and quantum knot invariants*

The main goal of this course is to describe an approach to construction of quantum knot invariants based on the combinatorial framework of triangulations. This approach is motivated by the the fact that comparing to knot diagrams triangulations present the following advantages:

- they offer the possibility of studying knots in arbitrary three-manifolds
- they display intrinsic three-dimensionality
- they make direct connection to the topology and geometry of knot complements.

Additionally, some 4-dimensional aspects of the considered constructions will also be discussed.

Plan:

1. Triangulations and Pachner moves.
2. Shaped triangulations.
3. H-triangulations.
4. Definition of the partition function of a shaped triangulation within Teichmüller TQFT.
5. Examples of calculation.

**Patrick Popescu Pampu** (Université Lille I)

*Contact topology and singularities*

This course will be an introduction to the study of interactions between singularity theory of complex analytic varieties and contact topology. We will concentrate on the relation between the smoothings of singularities and the Stein fillings of their contact boundaries.

**Takuya Sakasai** (University of Tokyo)

*Johnson-Morita theory*

The role of the mapping class group and its Torelli subgroup in low dimensional topology is widely accepted to be important and structures of these mysterious groups have been studied for a long time.

In 1980's, Dennis Johnson introduced a homomorphism, now called the Johnson homomorphism,from the Torelli subgroup to a symplectic module and used it to prove several fundamental (but deep) facts on the Torelli subgroup.

After that, Shigeyuki Morita clarified and extended this theory. In a series of his works, he revealed close relationships to invariants of 3-dimensional manifolds and cohomology of the mapping class group. Since then, this theory has been further extended by many people in various directions.

In this series of talks, starting from the review of works of Johnson and Morita, we discuss recent developments of the theory of Johnson homomorphisms and its applications:

1. Mapping class group, Torelli subgroup and the first Johnson homomorphism
2. Higher Johnson homomorphisms
3. Extensions and Applications

## Short talks

**Ben Aribi Fathi** (Univ. de Genève)

*$L^2$-Alexander invariants in knot theory*

The $L^2$-Alexander invariant of a knot is a continuous real function defined by Li and Zhang in 2006 as an infinitedimensional version of the Alexander polynomial. It contains classical invariants of geometry and topology like the simplicial volume and the genus. Using 3-dimensional topology, we will explain how this invariant detects several knots among the set of all knots.

**Celoria Daniele** (Univ. de Firenze)

*Grid homology in lens spaces*

After a short introduction on grid homology for knots in lens spaces, we will give some application to concordances and rational genus.

**Chavli Eirini** (Univ. d'Amiens)

*The irreducible representations of B3 of dimension at most 5*

In 1999 I. Tuba and H. Wenzl classified the irreducible representations of the braid group B3 on 3 strands of dimension at most 5, by giving explicit matrices of a triangular form whose coefficients are completely determined by the eigenvalues and a $r^{th}$ root of their determinant. Moreover, they proved that such irreducible representations exist if and only if the eigenvalues do not annihilate some polynomials in the eigenvalues and r. In this talk we will explain the reason we have matrices in this neat form, as well as, the origin of the above polynomials by reproving this classification as a consequence of the freeness conjecture for the generic Hecke algebra of the finite quotient of $B^3$.

**Conway Anthony** (Univ. de Genève)

*Colored tangles and signatures*

I will discuss signatures of links, relate these invariants to the Burau representation of the braid group and discuss twisted generalisations of these objects.

**Damiani Celeste** (Univ. de Caen)

*On the Alexander polynomial of a ribbon tangle*

Ribbon 2-knotted objects are locally flat 2-dimensional submanifolds of R4 that bound immersed 3-manifolds with only ribbon singularities. They appear as topological realizations of welded knotted objects, where welded knot theory is a quotient of virtual knot theory. We construct a functorial extension of the Alexander polynomial to ribbon tangles. At a combinatorial level this gives rise to a generalization of the Alexander polynomial of links to welded tangles.

**Detcherry Renaud** (Michigan State Univ.)

*Asymptotics of curve operators in TQFT*

Curve operators are endomorphisms associated to curves on a curve by Reshetikhin-Turaev TQFTs. We express the asymptotic of their matrix coefficients in terms of trace functions on the moduli space.

**Kohli Ben-Michael** (Univ.de Bourgogne)

*Some interesting evaluations of the Links-Gould polynomials*

The Links-Gould polynomials $LG^{m;n}(t_0; t_1)$ are two variable polynomial link invariants. Each one of them is derived from a highest weight representation of quantum supergroup $U_q(gl(mjn))$. In 2005, De Wit, Ishii and Links proved that the Alexander-Conway polynomial of a link could be recovered as an evaluation of certain of these Links-Gould polynomials. I shall highlight another specialization that allows us to recover powers of the Alexander invariant, expressing representations of the braid groups derived from the R-matrices of $LG^{m;n}$ in terms of Burau representations. These evaluations together with tests on values of LG for small prime knots encourage us to believe that the Links-Gould invariant should have a classical interpretation, in quite the same manner as the Alexander polynomial has.

**Moussard Delphine** (Univ. de Bourgogne)

*Braid group orbits in Aff(C)-character varieties of the punctunred sphere*

Joint work with Gaël Cousin. The group $PBn(S^2)$ of n-strands pure braids of the sphere acts naturally on the representations of the fundamental group of the n-punctured sphere. Ga¨el Cousin has shown that finite orbits of such actions provide interesting flat connections on vector bundles over projective ruled varieties. Motivated by this result, we consider the representations of the fundamental group of the n-punctured sphere in the complex affine group. We will describe the finite orbits of the action of the group $PBn(S^2)$ on these representations.

**Morvan Xavier** (Univ. de Genève)

*Non-commutative generalisation of Thurston's gluing equations*

In his famous Princeton Notes, Thurston introduces the so-called gluing equation, defined from ideal triangulations of knot complements. Those are algebraic equations with complex parameters encoding the hyperbolic structures of the knot complement. In this talk, we will define algebraically a non-commutative generalisation of those equation from H-triangulation of the couple ($S^3$;K).

**Stylianakis Charalampos** (Univ. of Glasgow)

*The normal closure of a power of a half-twist has infinite index in the mapping class group of the punctured sphere*

The Jones representation of the mapping class group of the punctured sphere is constructed by formulating irreducible linear representations of braid groups that factor through Hecke algebras. In this talk we introduce the Jones representation and we show that the normal closure of the $m^{th}$ power of a half-twist has infinite index in the mapping class group of a punctured sphere. As a corollary we show that the normal closure of a power of a Dehn twist has infinite index in the hyperelliptic mapping class group of a closed surface of genus at least two.