

# WINTERBRAIDS XIV

Bordeaux  
February 3–6, 2025

	Monday	Tuesday	Wednesday	Thursday
09:00-10:00	Cumplido 1	Kjuchukova 2	Lanneau 2	Lanneau 3
10:00-10:30	Coffee	Coffee	Coffee	Coffee
10:30-11:30	Lanneau 1	Cumplido 2	Kjuchukova 3	Bagherifard/Chemin
11:30-14:00	Lunch	Lunch	Lunch	Gavazzi
14:00-15:00	Kjuchukova 1	Brugallé	Cumplido 3	12:00 Lunch
15:00-15:30	Di Prisa	Malech	Zhang	
15:30-16:00	Coffee break	Coffee break	Coffee break	
16:00-17:00	Haïoun/Haladjian	Flash talks+posters	Jouteur/Panda	

# MINI-COURSES AND SPECIAL LECTURE

## Maria Cumplido Cabello: Parabolic subgroups of Artin-Tits groups

Artin-Tits groups are deeply connected to braid groups, which are a fundamental example of Artin-Tits groups. These connections reveal a rich interplay between the combinatorial, geometric, and topological aspects of both group families. In particular, braid groups and their actions on configuration spaces and the curve complex provide insights into broader questions about Artin groups and their representations. Given an Artin-Tits group with its standard presentation, a standard parabolic subgroup is the subgroup generated by a subset of the group's generators. More generally, a parabolic subgroup refers to any conjugate of a standard parabolic subgroup. These subgroups are not only a natural part of the group's structure but also play a crucial role in understanding the algebraic and geometric properties of Artin-Tits groups and the complexes on which they act. In this mini-course, we will explore the importance of parabolic subgroups in studying Artin groups. We will also discuss the key techniques and ideas that have driven recent breakthroughs in the field, including tools from combinatorics, topology, and geometric group theory. The goal is to provide a comprehensive picture of how parabolic subgroups and related constructions have shaped our understanding of Artin groups and their rich mathematical landscape.

## Erwan Lanneau: Pseudo-Anosov maps and their expansions factors

Pseudo-Anosov maps play an important role in the study of the geometry and dynamics of moduli spaces and mapping class groups, such as braid groups. A pseudo-Anosov map has an expansion factor (or dilatation), which records the exponential growth rate of the lengths of the curves under iterations. Thurston's train track theory relates the dynamics of pseudo-Anosov maps to that of Perron-Frobenius matrices: their dilatation is then the Perron-Frobenius eigenvalue. Less well known is the Rauzy-Veech induction, which is a powerful machinery for computing dilatations. In this talk, we will first explain some classical constructions of pseudo-Anosov maps (such as the Thurston-Veech construction). We will then look at the Rauzy-Veech induction and use it to get several applications.

## Alexandra Kjuchukova: Knot invariants from branched covers

Let  $K$  be a knot in  $S^3$ . Many classical invariants of  $K$  are defined using covers of  $S^3$  branched along  $K$ . Even more can be learned about  $K$  by extending such covers to branched covers of  $B^4$ ; or by showing that extensions do not exist. I will survey some results which arise in this context, with a focus on sliceness obstructions. Given a knot  $K$  equipped with a dihedral quotient of its group,  $\rho : \pi_1(S^3 \setminus K) \rightarrow D_p$ , I will give a necessary and sufficient condition for the existence of a smooth surface  $F \subset B^4$  with  $\partial F = K$  such that  $\rho$  extends to a map  $\pi_1(B^4 \setminus F) \rightarrow D_p$ . When such a surface exists, the signature of an associated branched cover of  $B^4$  gives rise to a knot invariant,  $\Xi_p(K, \rho)$ . I will prove that  $\Xi_p(K, \rho)$  can obstruct  $K$  from bounding, in alternate scenarios, either ribbon or slice disks in  $B^4$ . As a concrete application, I will show how, by an elementary examination of the Seifert form,  $\Xi_p(K, \rho)$  can be used to obstruct ribbonness for a family of algebraically slice knots. Time permitting, I will also relate  $\Xi_p(K, \rho)$  to other, old and new, knot invariants.

## Erwan Brugallé:

### **Enumeration of rational curves in algebraic surfaces or symplectic 4-varieties**

Over the last 20 years, considerable progress has been made in the study of the enumerative geometry of rational curves in algebraic or symplectic varieties, particularly in the case of real curves. In this talk, I will present methods for degenerating/cutting ambient varieties that are particularly effective in dimension 2 (complex) / 4 (real). The talk will focus on curves in surfaces. Time permitting, I will also discuss recent variations in algebraic geometry, over any field, of Gromov-Witten and Welschinger invariants.

## SHORT TALKS

### **Narges Bagherifard: Counting minimal tori in Riemannian manifolds**

The problem of counting closed geodesics in Riemannian Manifolds with negative curvature has been of interest for many years. Recently, Eftekhary has introduced a method for counting closed geodesics in closed manifolds with arbitrary curvature. Minimal surfaces in a Riemannian manifold are higher dimensional generalizations of closed geodesics. A natural question is how to count these types of submanifolds. We introduce a function that counts minimal tori in a Riemannian manifold  $(M, g)$  with  $\dim M > 5$ . Moreover, we show that this count function is invariant under perturbations of the metric.

### **Hugo Chemin: Cactus groups and remarkable subgroups**

Cactus groups  $J_n$  first appeared in the study of the structure of cobordism category by acting on tensorial products of objects of those categories. Then, they play an analogous role as braids groups in the braided categories. Moreover they appear in the study of operads. Indeed, there is a surjective morphism from  $J_n$  to the symmetric group  $S_n$  whose kernel is the group of pure cactus  $PJ_n$ , which is the fundamental group of the Deligne-Mumford compactification of moduli spaces of curves of real genus 0 with  $n + 1$  marked points. In this presentation I will start by describe cactus groups and some of their subgroups. In particular, I will use a result of J. Mostovoy to give some properties of these groups.

### **Alessio Di Prisa: On rational sliceness of negative amphichiral links**

We say that a link  $L$  in the 3-sphere is negative amphichiral if there exists an orientation-reversing diffeomorphism of the 3-sphere that sends every component of  $L$  to itself with the opposite orientation. If such a map can be chosen to be an involution, then the link is said to be strongly negative amphichiral. Kawauchi proved that every strongly negative amphichiral link is rationally slice, i.e. it bounds a disjoint collection of disks in a rational homology 4-ball. In this talk, we prove that every negative amphichiral link is rationally slice, extending the aforementioned work of Kawauchi. Our proof relies on a careful analysis of the JSJ decomposition of the link complement of negative amphichiral links. This is joint work in progress with Jaewon Lee (KAIST, Daejeon) and Oguz Savk (CNRS, Nantes).

### **Federica Gavazzi: On Decomposability of Virtual Artin Groups and Virtual Braids**

A group is said to be decomposable if it can be expressed as a direct product of two proper subgroups and indecomposable otherwise. For instance, braid groups are indecomposable, whereas certain dihedral groups are not. In this talk, we examine this property in the context of virtual Artin groups, a generalization of Artin groups that includes virtual braid groups. Specifically, we establish that virtual Artin groups are always indecomposable. As a consequence, we show that understanding the automorphism group of a virtual Artin group reduces to analyzing the automorphism groups of its irreducible components.

**Benjamin Haïoun:**

**Non-semisimple Witten-Reshetitkin-Turaev TQFTs at the boundary of Crane-Yetter**

I will review the very nice story due to Walker of how to find Witten-Reshetikhin-Turaev 3-dimensional Topological Quantum Field Theories at the boundary of Crane-Yetter 4-dimensional ones. Unfortunately, very few parts of this story are proven theorems, and I will discuss the gaps, proof strategies and non-semisimple generalizations.

**Igor Haladjian: Necklace groups and circular groups**

In this talk, we introduce torus necklaces, a family of links which almost coincide with Seifert links. We give a correspondence between their link groups and circular groups. Moreover, we briefly explain a correspondence between rank two complex braid groups and some necklace groups, as a particular case of a general correspondence with  $J$ -reflection groups.

**Perrine Juteur: Burau representation of  $B_4$  and  $q$ -deformed rational projective plane**

The braid group  $B_4$  naturally acts on the rational projective plane  $\mathbb{P}^2(\mathbb{Q})$ , this action corresponds to the classical integral reduced Burau representation of  $B_4$ . In this talk, I will first present a classification of the orbits for this action, and a description of the associated stabilizers. Then the Burau representation defines an action of  $B_4$  on  $\mathbb{P}^2(\mathbb{Z}(q))$ , where  $q$  is a formal parameter and  $\mathbb{Z}(q)$  is the field of rational functions in  $q$  with integer coefficients. The orbits through  $B_4$ -action in  $\mathbb{P}^2(\mathbb{Z}(q))$  will be considered as quantizations (or  $q$ -deformations) of the previous orbits. We will study these quantizations and show existence of embeddings of the  $q$ -deformed projective line  $\mathbb{P}^1(\mathbb{Z}(q))$  in the  $q$ -deformed projective plane that precisely correspond to the notion of  $q$ -rationals due to Morier-Genoud and Ovsienko.

### **Oliviero Malech: Internal and external stabilization of exotic surfaces in 4-manifolds**

In a smooth 4-manifold, two knotted surfaces can be topologically isotopic but not smoothly isotopic, forming what is known as an exotic pair. In this talk, we will address the following questions: What operations can be performed on an exotic pair to make them smoothly isotopic? And, is enlarging the ambient space sufficient to achieve this? Two main operations are commonly considered in the literature. The first, called internal stabilization, involves adding a tube to each embedded surface. The second, known as external stabilization, enlarges the ambient space by forming a connected sum with either  $S^2 \times S^2$  or  $CP^2 \# -CP^2$ . We will demonstrate a relationship between these two types of stabilization, and as an application, we will show that many examples of exotic surfaces become smoothly isotopic after precisely one external stabilization.

### **Pallavi Panda: Topology of some finite arc complexes**

The aim of the talk is to discuss the simplicial topology of some small non-compact crowned hyperbolic surfaces decorated with horoballs.

### **Butian Zhang: New combinatorial one-cocycles of space of long knotss**

The space of long knots has been studied from various perspectives. In this talk, we focus on the first cohomology classes of this space. Using Gauss diagrams, we construct two new independent combinatorial 1-cocycles that are practical to compute. Their values on specific loops in the space of long knots will also be calculated, yielding Vassiliev invariants.