WINTER BRAIDS V
School on braids and low-dimensional topology

Université de Pau
16–19 February, 2015

Organizing committee:
P. Bellingeri (Caen), V. Florens (Pau), J.B. Meilhan (Grenoble), E. Wagner (Dijon)

- Programme -

	Monday 16th	**Tuesday 17th**	**Wednesday 18th**	**Thursday 19th**
9:00	*Registration*	Kitano I	Dehornoy III	Costantino III
9:30	González-Meneses I			
10:00		*Coffee break*	*Coffee break*	*Coffee break*
10:30	*Coffee break*	González-Meneses II	Costantino II	Kitano III
11:00	Dehornoy I			
11:45		Gainullin	Kitano II	Casteluber
12:15	Misev	Aguilera		Barthel
12:45	*Lunch*	*Lunch*	*Lunch*	
14:15	Costantino I	Dehornoy II	González-Meneses III	
15:15	*Coffee break*	*Coffee break*	*Coffee break*	
15:45	Wedrich	Santharoubane	Chterental	
16:15	Manfredi	Silvero	Gobet	

- Abstracts -

Mini courses

François Costantino (Univ. Toulouse III)
TQFTs and quantum representations of mapping class groups

Despite their long name, Topological quantum field theories (TQFTs) admit a very short and clear definition, due to Atiyah.
Furthermore they have various topological applications, ranging from the construction of invariants of manifolds and knotted objects in manifolds to that of finite dimensional representations of mapping class groups.
In this course, we will first discuss the definition and some way of building TQFTs (with special attention to the universal construction) and then, after reviewing some general facts about mapping class groups, we will describe some of the main known examples of TQFTs.

Pierre Dehornoy (IF, Univ. Grenoble)
Knot theoretical invariants of 3-dimensional vector fields

In at least two distinct physical contexts arise some vector fields that are not constant over time, but are transported by diffeomorphism. It is then natural to look for invariants of vector fields under diffeomorphism--a question reminiscent of the quest for invariants of knots under diffeomorphism of the underlying space. A first example is the helicity that was introduced by Moffatt in the 50's. It was given a knot-theoretical definition by Arnold in the 70's as an "asymptotic linking number". Since then other knot invariants have been adapted to vector fields, but with a limited success: either the invariants are very hard to estimate or they are actually proportional to the helicity.
In this course, we will come back on the history of this problem, introduce certain invariants and explain why many of them are proportional to the helicity. Finally, we will sketch some recent hopes concerning this question.

Teruaki Kitano (Soka Univ., Tokyo)
Twisted Alexander polynomials

In this series of lectures we present an introduction for twisted Alexander polynomials of knots. It is not intended for experts. Though there are several versions of this invariant, we follow Wada's definition here. In general it can be defined for a finitely presentable group with an epimorphism onto a free abelian group. We consider only the knot case for simplicity in these lectures.
First we start from the definition of Alexander polynomial by using Fox's free differentials. Next we see how to generalize this definition for twisted cases. Further we show computations for some concrete examples and discuss fundamental properties. Finally we want to show one application for the existence of an epimorpshim between knot groups. We discuss the existence of an epimorphism implies the divisibility of twisted Alexander polynomials and its application, or some related topics.

Juan Gonzalez-Meneses (Univ. Sevilla)
Geometric approaches to Artin Tits groups

One interesting tool to determine properties of groups, comes from studying an action of the group on a space. Geometric properties of the space, like being a metric space, or more precisely a hyperbolic space, allow to use geometric techniques to study the group.
In this course we will survey some natural actions of braid groups and Artin-Tits groups on particularly nice spaces: On one hand, braids are mapping classes acting on (hyperbolic) surfaces, and this induces an action on a finite-dimensional simplicial complex, caled the complex of curves, which was shown to be hyperbolic by Masur and Minsky. On the other hand, Artin-Tits groups of spherical type act on a simplicial complex defined by Bestvina, and related to Garside normal forms, which allowed to determine some properties of these groups.
The course pretends to be an introduction to these topics, as self-contained as possible, not requiring any prior knowledge of Artin-Tits groups, hyperbolic spaces or Garside normal forms.

Short talks

Marta Aguilera (Univ. Sevilla)
Train tracks of rigid braids

The braid group acts on the curve complex of the n-times punctured disc. If a braid is pseudo-Anosov, its action has a north-south dynamic where the attracting limit point corresponds with the stable folitation. Bestvina and Hendel gave an algorithm to compute a graph, called train track, which encodes the foliation and the action of the braid. Their algorithm works for mapping class groups in general.
We will show how one can compute a train track in a different way, in the particular case of (so-called) rigid braids. As every pseudo-Anosov braid has a conjugate one of whose powers is rigid, this allows to compute a train track related to any pseudo-Anosov braid. Our aim is to explain how, using the Garside structure of braid groups, one can show that rigid braids admit particularly simple train tracks.

Senja Barthel (Imperial College, London)
Braids in Crystals

Coordination polymers (think of crystals) can have different topological modes. The ones of 1 dimensional coordination polymers can be described by braids. We predict all possible entanglements in those molecules. This is joint work with Davide M. Proserpio, F. Din-Houn Lau and Igor Baburin.

Vinicius Casteluber Laass (Sao Paolo)
The Borsuk-Ulam problem for homotopy class of functions - An approach using braid groups

The well-known Borsuk-Ulam theorem states that given any continuous map $f : S^2 \to R^2$, there is a point x in S^2 such that $f(-x) = f(x)$. One possible generalization of this theorem is to consider other spaces and involutions. If $\tau : M \to M$ is a free involution, we say that a triple $(M,\tau;N)$ has the Borsuk-Ulam property if for every continuous function $f : M \to N$, there is a point $x \in M$ such that $f(\tau(x)) = f(x)$. So if $(M,\tau;N)$ does not have the Borsuk-Ulam property, there exists a continuous map $f : M \to N$ such that $f(\tau(x)) \neq f(x)$ for every x in M. A natural question that arises is the following : up to homotopy, how many functions are there with this property ? In this talk, I will show a relation between this problem and a diagram involving braid groups, in the case that M and N are compact surfaces without boundary, and I will give some examples.

Oleg Chterental (Univ. Toronto)
Virtual braids and virtual curve diagrams

We define a faithful action of the virtual braid group VBn on certain planar diagrams called virtual curve diagrams. Our action is similar in spirit to the Artin action of the braid group Bn on the free group Fn and it provides an easy combinatorial solution to the word problem in VBn.

Fyodor Gainullin (Imperial College, London)
Heegaard Floer homology, Alexander polynomial and Dehn surgery on alternating knots

One of the biggest challenges in modern low-dimensional topology is to understand what knots can yield a given 3-manifold by Dehn surgery.
There do exist 3-manifolds, which can be obtained by Dehn surgery on infinitely many distinct knots. However, it turns out, that for every 3-manifold there are only finitely many alternating knots that can give it by surgery. I will attempt to go through the main steps of the proof, giving a rudimentary description of relevant tools from Heegaard Floer homology.

Thomas Gobet (TU Kaiserslautern)
Dual braid monoids and Hecke algebras

An understanding of the Hecke algebras using dual braid monoids would be of interest, especially for the complex reflection groups, since there is no canonical positive braid monoid. On the other hand, still in the Coxeter case, nothing has been done in this direction. We will review what is already known in the topic and give new recent results.

Enrico Manfredi (Univ. Bologna)
Links in lens spaces with inequivalent lift

A strong geometric invariant of links in lens spaces is the lift, that is to say a link in the 3-sphere that is the pre-image of a link under the universal cover of the lens space; the lift is clearly a freely periodic link. An interesting question is whether the lift is a complete invariant of links in lens spaces. Several counterexamples to this question are produced using braids. A class of links in lens spaces can be easily described by a braid, and so their lift. A short tabulation of the possible lifts gives two pairs of links with equivalent lift. Moreover cabling one of these examples with palindromic braids produces an infinite family with such a property.

Filip Misev (Univ. Bern)
Cutting and gluing fibre surfaces

A classical theorem in the theory of fibred links states that the fibering Seifert surfaces of any two such links in the three-sphere are related by a sequence of so-called Hopf plumbings and deplumbings. The aim of the talk is to explore this relation and to give examples that illustrate how (non-)unique such plumbing sequences can be.

Ramanujan Santharoubane (Paris VII)
On the AMU conjecture for the four holed sphere

We are going to focus on quantum representations of the mapping class group of the four holed sphere arising from the Witten-Reshetikhin-Turaev SU(2) Topological Quantum Field Theory (TQFT). We will see how to extend the result concerning the asymptotic behavior of pseudo-Anosov elements proved by J.E Andersen, G. Masbaum and K. Ueno.

Marithania Silvero (Univ.Sevilla)
Positivity of Conway polynomials of closed BKL-positive 3-braids

In 1989, Peter Cromwell proved that positive links have positive Conway polynomial; that is, all the coefficients are non-negative. Positive links include those links which are closure of positive braids in terms of Artin generators. A link is said to be BKL-positive if it can be expressed by a positive braid word using the generators introduced by Birman, Ko and Lee in 1998. Not every BKL-positive link is positive. In this talk we show that BKL-positive links with braid index 3 have positive Conway polynomial.

Paul Wedrich (Univ. Cambridge)
Deformations of link homologies

I will start by explaining how deformations help to answer two important questions about the family of (colored) sl(N) link homology theories: What geometric information about links do they contain? What relations exist between them? I will recall results of Lee, Gornik and Wu on generic deformations of sl(N) link homologies and sketch how they generalize to include the case of non-generic deformations. The result is a decomposition theorem for deformed colored sl(N) link homologies which leads to new spectral sequences between various type A link homologies and concordance invariants in the spirit of Rasmussen's s-invariant. Joint work with David Rose.